Stability of a Dual-Spin Satellite with Two Dampers

KYLE T. ALFRIEND*
NASA Goddard Space Flight Center, Greenbelt, Md.
AND
CARL H. HUBERT†
Cornell University, Ithaca, N.Y.

An investigation is made of the stability of a dual-spin satellite with a spring-mass damper on one portion and a partially filled viscous ring damper on the other portion. Time constants are obtained from the approximate equations of motion and then compared with the time constants from numerical integration of the exact equations of motion. For certain configurations, it is shown that a limit cycle can exist.

Introduction

HOLLOW rings, partially filled with fluid, are sometimes employed on satellites as heat pipes. The motion of the fluid in such a pipe, however, is a source of energy dissipation, and as such, affects the rotational stability of the system. The partially filled viscous ring damper was first studied by Carrier and Miles,[1] who demonstrated that the fluid begins to behave as a lumped mass or slug of fluid above very small nutation angles. The next analysis, performed by Cartwright, Massingill, and Trueblood,[2,3] assumed that the fluid mass behaved as a particle moving in a tube with a viscous damping force. Alfriend[4] used the same approach to study the single-spin system, with good results.

NASA has contemplated mounting rings partially filled with fluid on the rapidly spinning portion of some dual-spin satellites for use as heat pipes during despun portions of the mission. The purpose of this study is to determine the effect of the motion of the fluid in these rings on the rotational stability. The fluid is treated as a particle moving in a tube with viscous damping. An additional passive damper system is considered to be mounted on the slowly spinning portion of the satellite. A single mass-spring damper is chosen for this purpose primarily for simplicity, but the results can readily be extended to other types of dampers and to multiple dampers. Only one viscous ring damper is used in this analysis, but once again, the results can readily be extended to multiple dampers.

Since the equations of motion of the system are highly nonlinear, the criteria for rotational stability are determined through the use of approximate solutions. The results are compared with those obtained through numerical integration of the equations of motion.

Description of the System

Consider the dual-spin satellite shown in Fig. 1, consisting of a main body and a symmetric rotor, both of which spin about a common axis. Attached to the main body is a mass-spring-dashpot damper, aligned with and a distance b from the spin axis. The spring and damping constants are k and c_d, respectively, and the mass is $\mu_d M$, where M is the total mass of the system and μ_d is a constant of proportionality. The displacement of the spring from its free length position is z. Attached to the rotor is a viscous ring damper of radius r, which is centered at and perpendicular to the spin axis. The damper contains a quantity of fluid, assumed to behave as a particle, with a total mass $\mu_s M$. The damping constant is c_s.

Let X, Y, and Z be the principal axes, and $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ the associated unit vectors of the system for which $r = z = 0$. When $z = 0$, the damper mass lies on the X axis. The viscous ring damper lies a distance l above the XY plane. The principal moments of inertia are I_x, I_y, and I_z. The rotor has a moment of inertia about the Z axis of I_R.

The main body of the satellite rotates with respect to the inertial frame with an angular velocity $\boldsymbol{\omega} = \omega_x \mathbf{e}_x + \omega_y \mathbf{e}_y + \omega_z \mathbf{e}_z$. The rotor rotates with respect to the main body with an angular velocity $\dot\phi \mathbf{e}_z$. The main body exerts a torque T_R on the rotor, along the Z axis. The Z axis is the intended spin axis.

Problem Solution

The equations of motion for the system are developed in the appendix. Since $\mu_d \ll 1$ and $\mu_s \ll 1$, approximate equations are obtained by dropping all terms of $0(\mu_d)$ and $0(\mu_s)$. It is generally desired that the slowly rotating portion of the satellite have a constant angular velocity. It is assumed that this state is achieved through the use of an internal control system which causes T_r to be such that $\omega_z = \Omega$ (a constant). The approximate equations of motion are

$$0 = I_x \dot\omega_x + (I_z - I_y)\omega_y \Omega + I_r \dot\phi \omega_y \quad (1)$$

$$0 = I_y \dot\omega_y + (I_x - I_z)\omega_x \Omega - I_r \dot\phi \omega_x \quad (2)$$

$$0 = (I_y - I_x)\omega_x \omega_y + I_r \ddot\phi + c_s r^2(\dot\phi - \dot\psi) \quad (3)$$

$$0 = M\ddot z + D_d \dot z + [K - M(\omega_x^2 + \omega_y^2)]z + Mb(\omega_x \Omega - \dot\omega_y) \quad (4)$$

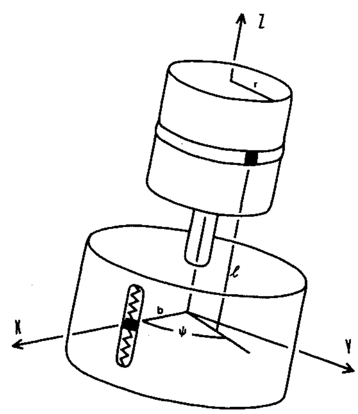

Fig. 1 Mathematical model and coordinate system.

Received December 4, 1973; revision received March 21, 1974.
Index category: Spacecraft Attitude Dynamics and Control.

* NRC/NAS Senior Postdoctoral Resident Research Associate. Member AIAA.

† Graduate Student, Department of Theoretical and Applied Mechanics. Member AIAA.

$$0 = Mr\ddot{\psi} + D_s r(\dot{\psi} - \dot{\phi}) + Mr\omega_x\omega_y(\cos^2\psi - \sin^2\psi) + Mr(\omega_y{}^2 - \omega_x{}^2)\sin\psi\cos\psi - Ml(\dot{\omega}_y + \omega_x\Omega)\sin\psi - Ml(\dot{\omega}_x - \omega_y\Omega)\cos\psi \quad (5)$$

$$0 = I_r\ddot{\phi} + c_s r^2(\dot{\phi} - \dot{\psi}) - T_r \quad (6)$$

where $D_s \equiv c_s/\mu_s$, $D_d \equiv c_d/\mu_d$, and $K \equiv k/\mu_d$. From Eqs. (3) and (6) it is seen that

$$T_r = (I_x - I_y)\omega_x\omega_y$$

Assuming the satellite to be inertially symmetric (i.e., $I_x = I_y = I_t$), and assuming $\dot{\phi}$ to be nearly constant, permits Eqs. (1) and (2) to be expressed as

$$0 = \dot{\omega}_x + \lambda\omega_y$$

$$0 = \dot{\omega}_y - \lambda\omega_x$$

where

$$\lambda \equiv [(I_z - I_t)\Omega + I_r\dot{\phi}]/I_t$$

This leads immediately to

$$\omega_x = A\cos(\lambda t + \beta) \quad (7)$$

$$\omega_y = A\sin(\lambda t + \beta) \quad (8)$$

where A and β are constants of integration. It is seen that $(\lambda t + \beta)$ is the approximate angular position of the plane defined by the spin axis (Z axis) and the total angular momentum vector. This plane will be referred to as the "nutation plane."

The nutation angle Φ is defined by

$$\tan\Phi \equiv H_t/H_z \quad (9)$$

where $H_t \equiv (H_x{}^2 + H_y{}^2)^{1/2}$. This is evaluated using the angular momentum equations in the Appendix. Neglecting terms of $0(\mu_d)$ and $0(\mu_s)$, and substituting for ω_x and ω_y, produces the approximation

$$A = (\lambda + \Omega)\tan\Phi \quad (10)$$

Differentiation of Eq. (9) leads to

$$\dot{\Phi} = -\dot{H}_z/H_t \quad (11)$$

where $\dot{H}_z = \omega_y H_x - \omega_x H_y$. Expanding, and discarding terms which are nonlinear in μ_d and μ_s, yields the approximate equation for the time rate of change of the nutation angle

$$\dot{\Phi} = (M/I_t)\{\mu_d b[z\Omega\sin(\lambda t + \beta) - \dot{z}\cos(\lambda t + \beta)] - \mu_s r^2(\lambda + \Omega)\tan\Phi\cos(\psi - \lambda t - \beta)\sin(\psi - \lambda t - \beta) - \mu_s rl(\dot{\psi} + \Omega)\sin(\psi - \lambda t - \beta)\} \quad (12)$$

Substituting for ω_x and ω_y in Eq. (4), the equation of motion of the damper mass becomes

$$\ddot{z} + (D_d/M)\dot{z} + [(K/M) - (\lambda + \Omega)^2\tan^2\Phi]z = b(\lambda^2 - \Omega^2)\tan\Phi\cos(\lambda t + \beta)$$

Considering Φ to be slowly varying leads to the approximate particular solution

$$z = [b(\lambda^2 - \Omega^2)/(\alpha_1{}^2 + \alpha_2{}^2)]\tan\Phi[\alpha_1\cos(\lambda t + \beta) + \alpha_2\sin(\lambda t + \beta)] \quad (13)$$

where

$$\alpha_1 = K/M - \lambda^2 - (\lambda + \Omega)^2\tan^2\Phi \quad (14)$$

$$\alpha_2 = (D_d/M)\lambda \quad (15)$$

The homogeneous solution vanishes with time and is ignored. It should be noted that discarding terms of $0(\mu_d)$ and $0(\mu_s)$ in obtaining the approximate equations of motion, decoupled the spring-mass damper motion from the fluid particle motion.

At this point, attention is directed to the fluid particle. The motion of the particle in the ring is essentially governed by the effects of viscous friction and centripetal acceleration. For sufficiently small nutation angles, viscous friction predominates, and after transients die out, the particle rotates with the ring. The centripetal force, however, increases with increasing nutation angle, eventually predominates, and causes the particle to rotate with the nutation plane. Cartwright, Massingill, and Trueblood,[2] referred to the first motion as the "spin-synchronous" mode, and the second motion as the "nutation-synchronous" mode.

Nutation-Synchronous Mode

Let α represent the angular position of the fluid particle with respect to the nutation plane. Since $(\lambda t + \beta)$ is the approximate position of the nutation plane

$$\alpha = \psi - \lambda t - \beta \quad (16)$$

Substitution into Eq. (5) leads to

$$0 = Mr\ddot{\alpha} + D_s r(\dot{\alpha} + \lambda - \dot{\phi}) - M(\lambda + \Omega)^2[l + r\tan\Phi\cos\alpha] \times \tan\Phi\sin\alpha$$

If $\dot{\phi}$ is considered to be a constant and Φ is considered to be slowly varying, an approximate solution to this equation is the α which satisfies

$$D_s r(\lambda - \dot{\phi}) = M(\lambda + \Omega)^2[l + r\tan\Phi\cos\alpha]\tan\Phi\sin\alpha \quad (17)$$

This solution represents the nutation-synchronous mode.

Substitution of Eqs. (13, 16, and 17) into Eq. (12), leads immediately to

$$\dot{\Phi} = [M\mu_d b^2(\lambda^2 + \Omega^2)\tan\Phi/I_t(\alpha_1{}^2 + \alpha_2{}^2)] \times \{\alpha_1(\lambda + \Omega)\sin(\lambda t + \beta)\cos(\lambda t + \beta) + \alpha_2[\Omega\sin^2(\lambda t + \beta) - \lambda\cos^2(\lambda t + \beta)]\} + \mu_s D_s r^2(\dot{\phi} - \lambda)/I_t(\lambda + \Omega)\tan\Phi$$

It is assumed that the nutation angle is small enough to permit the approximation $\tan\Phi = \Phi$. Defining $\gamma = \Phi^2$, the equation can be rewritten as

$$d\gamma/d\tau + (R\sin\tau + S)\gamma = B \quad (18)$$

where

$$R = \lambda Q[(\alpha_1{}^2 + \alpha_2{}^2)(\lambda + \Omega)^2]^{1/2} \qquad S = \lambda Q(\Omega - \lambda)$$

$$Q = 2M\mu_d b^2(\Omega^2 - \lambda^2)/I_t(\alpha_1{}^2 + \alpha_2{}^2) \qquad \tau = 2(\lambda t + \beta + v)$$

$$B = 4\lambda\mu_s D_s r^2(\dot{\phi} - \lambda)/I_t(\lambda + \Omega) \qquad v = \tfrac{1}{2}\tan^{-1}(B/2\lambda Q)$$

Equation (18) has the solution

$$\gamma = \exp(R\cos\tau - S\tau)[B\textstyle\int\exp(S\tau - R\cos\tau)\,d\tau + P_o]$$

where P_o is a constant of integration. The integral can be evaluated by expanding $\exp(-R\cos\tau)$ in powers of $(-R\cos\tau)$

$$\int\exp(S\tau - R\cos\tau)d\tau = \sum_{n=0}^{\infty}\frac{1}{n!}\int\exp(S\tau)(-R\cos\tau)^n\,d\tau = \exp(S\tau)\sum_{i=0}^{\infty}\sum_{n=i}^{\infty}\sum_{j=0}^{1}\frac{(-R)^{2n+j}(\cos\tau)^{2(n-i)+(j-1)}}{(2n-2i+j)!}\cdot\frac{[S\cos\tau + 2(n+i+\frac{1}{2}j)\sin\tau]}{\prod_{k=0}^{i}[S^2 + (2n-2k+j)^2]}$$

The series is absolutely convergent for all $\tau < \infty$. The solution of the differential equation becomes

$$\gamma = \exp(R\cos\tau)\left[P_o\exp(-S\tau) + B\sum_{i=0}^{\infty}\sum_{n=i}^{\infty}\sum_{j=0}^{1}\times\frac{(-R)^{2n+j}(\cos\tau)^{2(n-i)+(j-1)}}{(2n-2i+j)!}\cdot\frac{[S\cos\tau + 2(n-i+\frac{1}{2}j)\sin\tau]}{\prod_{k=0}^{i}[S^2 + (2n-2k+j)^2]}\right]$$

Since R is generally small, an approximate solution is

$$\gamma = P_o\exp(-S\tau) + B/S$$

Defining

$$C_1 \equiv -\frac{2\lambda}{S} = \frac{-I_t(\alpha_1{}^2 + \alpha_2{}^2)}{M\mu_d b^2\alpha_2(\lambda + \Omega)(\lambda - \Omega)^2} \quad (19)$$

$$C_2 \equiv B/2\lambda = 2\mu_s D_s r^2(\dot{\phi} - \lambda)/I_t(\lambda + \Omega) \quad (20)$$

and substituting for τ, finally leads to

$$\Phi = [P\exp(t/C_1) - C_1 C_2]^{1/2} \quad (21)$$

where P is a constant.

Equation (21) represents the approximate behavior of the nutation angle when the system is in the nutation-synchronous mode. It is noted that a limit cycle exists if $C_1 < 0$ and $C_1C_2 < 0$. In such a case, Φ approaches $(-C_1C_2)^{1/2}$ asymptotically from both above and below. The possibility of such limit cycle behavior was first demonstrated in 1970 by Likins, Tseng, and Mingori,[5] for the dual-spin system with cubic damping.

Alfriend,[6] investigated the partially filled viscous ring damper on a single-spin satellite, treating the fluid as a slug of finite length. He showed that the ring has an effective height equal to the product of the actual height with $[\sin(\gamma/2)]/(\gamma/2)$, where γ is the angle of fill. None of the other differences between the treatment of the fluid as a slug of finite length, and the treatment of the fluid as a particle, appear in the linearized equations of motion. It is seen that this result must hold for the dual spin system as well. Hence, replacing l with

$$l_{\text{effective}} = l \sin(\gamma/2)/(\gamma/2)$$

will extend the results to a more general case.

For small Φ, Eq. (17) becomes

$$\sin\alpha = D_s r(\lambda - \dot{\phi})/Ml(\lambda+\Omega)^2\Phi$$

As Φ decreases, α increases, and the fluid particle moves away from the nutation plane. When $\sin\alpha = \pm 1$, the particle begins to rotate with the ring and oscillate about a point within it. The approximate magnitude of Φ at which this transition to spin synchronous motion occurs is

$$\Phi_t = |D_s r(\dot{\phi} - \lambda)/Ml(\lambda+\Omega)^2|$$

Spin-Synchronous Mode

The behavior of the fluid particle in the spin-synchronous mode can be expressed by the approximation

$$\psi = \psi_o + \dot{\phi}t + f(t) \tag{22}$$

where ψ_o is a constant, $\dot{\phi}$ is assumed to be nearly constant, and $|f(t)| \ll 1$. Defining $\eta_1 \equiv (\dot{\phi}-\lambda)$ and $\eta_2 \equiv (\psi_o - \beta)$, and using the approximations $\tan\Phi \cong \Phi$ and

$$\sin[\eta_1 t+\eta_2+f(t)] \cong \sin(\eta_1 t+\eta_2)+f(t)\cos(\eta_1 t+\eta_2)$$

$$\cos[\eta_1 t+\eta_2+f(t)]\sin[\eta_1 t+\eta_2+f(t)] \cong \cos(\eta_1 t+\eta_2)\sin(\eta_1 t+\eta_2)+f(t)[\cos^2(\eta_1 t+\eta_2)-\sin^2(\eta_1 t+\eta_2)]$$

in Eq. (12), yields

$$\dot{\Phi} = (M/I_t)\{\mu_d b[z\Omega\sin(\lambda t+\beta)-\dot{z}\cos(\lambda t+\beta)] - \mu_s r^2(\lambda+\Omega)\Phi[\cos(\eta_1 t+\eta_2)\sin(\eta_1 t+\eta_2)+ f(t)\cos^2(\eta_1 t+\eta_2)-f(t)\sin^2(\eta_1 t+\eta_2)] - \mu_s rl(\dot{\psi}+\Omega)[\sin(\eta_1 t+\eta_2)+f(t)\cos(\eta_1 t+\eta_2)]\} \tag{23}$$

Expanding ω_x, ω_y, and $\dot{\psi}$ in Eq. (5), and linearizing Φ, yields the approximate differential equation of $f(t)$ for small nutation angles

$$\ddot{f}(t)+(D_s/M)\dot{f}(t) = (l/r)(\lambda+\Omega)^2\Phi[\sin(\eta_1 t+\eta_2)+f(t)\cos(\eta_1 t+\eta_2)]$$

Since $|f(t)| \ll 1$, this can be further reduced to

$$\ddot{f}(t)+(D_s/M)\dot{f}(t) = (l/r)(\lambda+\Omega)^2\Phi\sin(\eta_1 t+\eta_2)$$

which has the solution

$$f(t) = (G/\eta_1)(\lambda+\Omega)\Phi[\eta_1\sin(\eta_1 t+\eta_2)+(D_s/M)\cos(\eta_1 t+\eta_2)] \tag{24}$$

where

$$G = -M^2 l(\lambda+\Omega)/r(M^2\eta_1^2+D_s) \tag{25}$$

This solution for $f(t)$ is substituted into Eq. (23), and Eq. (13) is used to replace z and $\dot{z}$. Discarding those terms which are nonlinear in Φ yields

$$\dot{\Phi}+g(t)\Phi = h(t) \tag{26}$$

where

$$g(t) = K_1[\alpha_1(\Omega+\lambda)\sin 2(\lambda t+\beta)-\alpha_2(\Omega+\lambda)\cos 2(\lambda t+\beta)+ \alpha_2(\Omega-\lambda)]+K_2\left\{Gl\left(\frac{D_s(\Omega+\dot{\phi})}{M\eta_1}-\frac{D_s}{M}\right)+ Gl\left(\frac{D_s(\Omega+\dot{\phi})}{M\eta_1}+\frac{D_s}{M}\right)\cos 2(\eta_1 t+\eta_2)+ [r+Gl(\Omega+\dot{\phi}+\eta_1)]\sin 2(\eta_1 t+\eta_2)\right\} \tag{27}$$

$$h(t) = K_3\sin(\eta_1 t+\eta_2) \tag{28}$$

$$K_1 = M\mu_d b^2(\Omega^2-\lambda^2)/2I_t(\alpha_1^2+\alpha_2^2) \tag{29}$$

$$K_2 = M\mu_s r(\lambda+\Omega)/2I_t \tag{30}$$

$$K_3 = -M\mu_s rl(\Omega+\dot{\phi})/I_t \tag{31}$$

The general solution of Eq. (26) is

$$\Phi = \exp(-\gamma)[\textstyle\int\exp(\gamma)h(t)\,dt+K_4] \tag{32}$$

where K_4 is an integration constant and

$$\gamma = \int g(t)\,dt$$

Integrating $g(t)$ leads to

$$\gamma = (1/\tau_d+1/\tau_s)t-[K_1(\Omega+\lambda)/2\lambda][\alpha_1\cos 2(\lambda t+\beta)+ \alpha_2\sin 2(\lambda t+\beta)]+K_2\{(GlD_s/2M\eta_1^2)(\Omega+\dot{\phi}+\eta_1)\times \sin 2(\eta_1 t+\eta_2)-(1/2\eta_1)[r+Gl(\Omega+\dot{\phi}+\eta_1)]\cos 2(\eta_1 t+\eta_2)\} \tag{33}$$

where

$$\tau_d = 1/K_1\alpha_2(\Omega-\lambda) \tag{34}$$

$$\tau_s = M\eta_1/K_2 GlD_s(\Omega+\lambda) \tag{35}$$

It is seen that

$$C_1 = -\tau_d/2 \tag{36}$$

and

$$C_2 = -I_t[M^2(\dot{\phi}-\lambda)^2+D_s^2]/\tau_s[\mu_s rlMD_s(\lambda+\Omega)]^2 \tag{37}$$

It can be shown that the solution of

$$\int\exp(\gamma)h(t)\,dt$$

may be expressed as

$$\exp\left[\left(\frac{1}{\tau_d}+\frac{1}{\tau_s}\right)t\right]\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}\{a_{nm}\cos[2n(\lambda t+\beta)+ m(\eta_1 t+\eta_2)]+b_{nm}\sin[2n(\lambda t+\beta)+m(\eta_1 t+\eta_2)]\}$$

where a_{nm} and b_{nm} are constants. Therefore, the solution [Eq. (32)] is of the form

$$\Phi = \exp\left[\left(\frac{1}{\tau_d}+\frac{1}{\tau_s}\right)t-\gamma\right]\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}\{a_{nm}\cos[2n(\lambda t+\beta)+ m(\eta_1 t+\eta_2)]+b_{nm}\sin[2n(\lambda t+\beta)+m(\eta_1 t+\eta_2)]\}+ K_4\exp(-\gamma)$$

In a practical system, the mass of the mass-spring damper and the mass of the fluid particle will be sufficiently small so that, except at very small nutation angles, a_{nm}, b_{nm}, and the coefficients of the oscillatory terms in γ will be negligible. Hence, when the system is in the spin-synchronous mode, the behavior of the nutation angle can be approximated by the equation

$$\Phi = K_4\exp[-(1/\tau_d+1/\tau_s)t] \tag{38}$$

As with the nutation-synchronous mode, this result can be extended to the more general case, in which the fluid is treated as a slug of finite length, by replacing l in τ_s by

$$l_{\text{effective}} = l\sin(\gamma/2)/(\gamma/2)$$

where γ is the angle of fill.

Rotational Stability of the System

The system is considered to be rotationally stable if the nutation angle has a tendency to decrease, and rotationally unstable if it has a tendency to increase. Examination of Eqs. (21, 36, 37, and 38) shows that the behavior of Φ, and hence the stability of the system, falls into four distinct cases.

Case I ($\tau_d > 0$, $\tau_s > 0$)

Both the nutation-synchronous and spin-synchronous modes are stable. The energy dissipated by the motion of the fluid particle contributes to the stability. The numerical integration of a typical nutation angle decay of this type, including the transition from nutation-synchronous to spin-synchronous motion, is shown in Fig. 2.

Case II ($\tau_d > 0$, $\tau_s < -\tau_d$)

The spin-synchronous mode is stable. In the nutation-synchronous mode, the nutation angle approaches $(-C_1C_2)^{1/2}$ asymptotically, from both above and below. If $\Phi_o > (-C_1C_2)^{1/2}$,

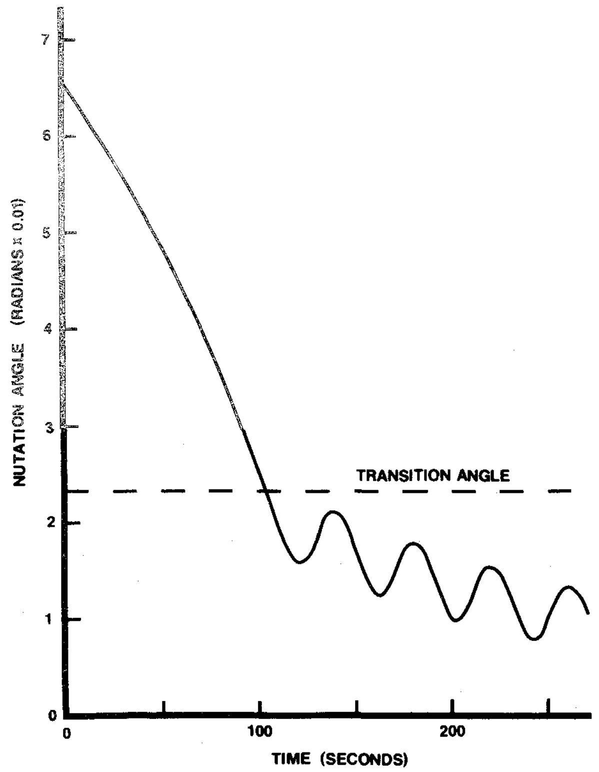

Fig. 2 Typical Case I nutation angle decay.

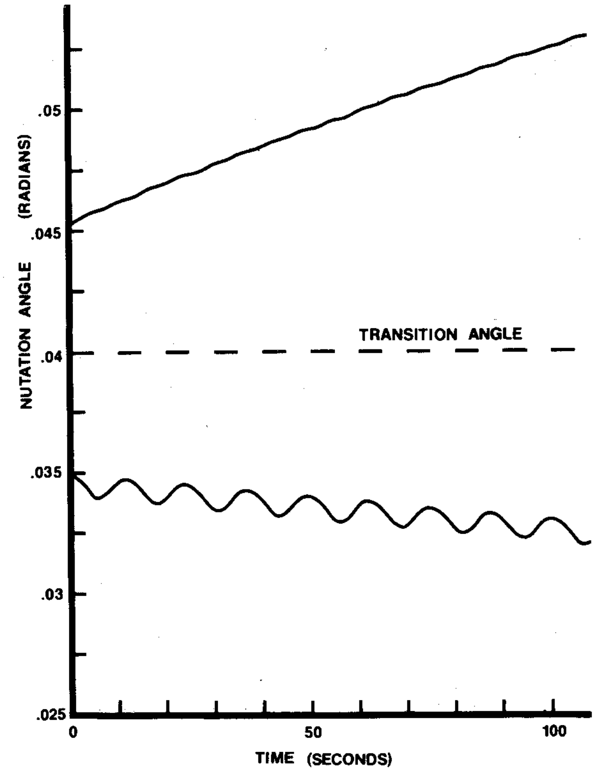

Fig. 4 Typical Case II behavior in the vicinity of the transition angle.

the steady state is a small oscillation about $\Phi = 0$. However, if $\Phi_t < (-C_1C_2)^{1/2}$, there are two possible steady states. One of these is the oscillation, the other is $\Phi = (-C_1C_2)^{1/2}$.

The motion of the fluid particle is a destabilizing influence on the system. However, below the transition angle and above the asymptote, the stabilizing influence of the spring-mass damper dominates, although the decay of the nutation angle is retarded.

Figure 3 shows the results of numerical integrations illustrating the asymptotic behavior of the nutation angle, and Fig. 4 shows the results of numerical integrations above and below the transition angle.

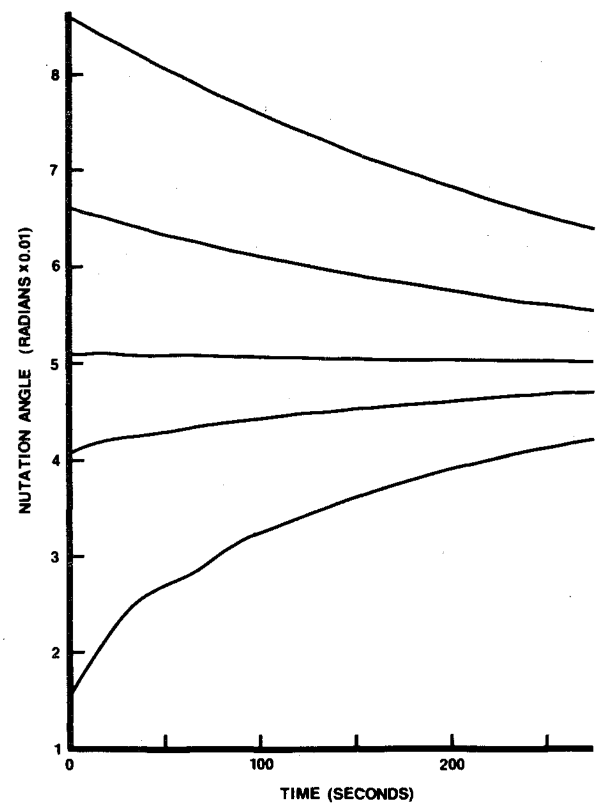

Fig. 3 Typical asymptotic behavior.

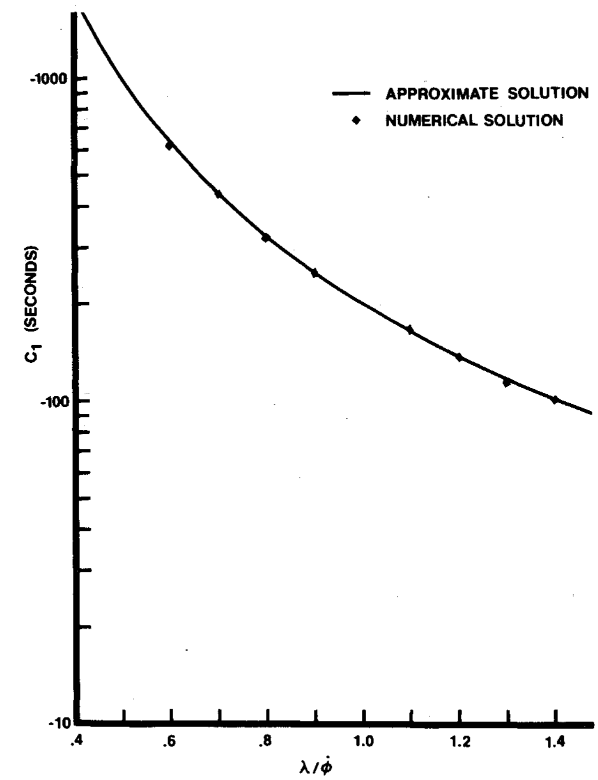

Fig. 5 Approximate vs exact values of C_1.

Case III $(\tau_d > 0, 0 > \tau_s > -\tau_d)$

The spin-synchronous mode is unstable, and the asymptotic behavior of the nutation-synchronous mode (discussed in Case II) is repeated here. The destabilizing influence of the fluid particle overwhelms the stabilizing influence of the mass-spring damper for nutation angles smaller than Φ_t or $(-C_1C_2)^{1/2}$, whichever is larger.

Case IV $(\tau_d < 0)$

The mass-spring damper is a destabilizing influence, and hence, this case is out of the realm of realistic design.

Results of Numerical Integration

The exact equations of motion were integrated using a fourth-order Runge-Kutta method. Comparison was made between the numerical (or "exact") solutions and the "approximate" solutions [Eqs. (21) and (38)], by assuming the forms of the "approximate" solutions to be correct, determining the constants from numerical solutions, and comparing the constants thus obtained with the predicted constants. Figures 5 and 6 compare the approximate and exact values of the spring-mass damper constant C_1 and the viscous ring damper constant C_2. The comparisons are favorable.

Figure 7 compares the predicted total time constant τ for the spin-synchronous mode, where

$$\tau = \tau_d \tau_s/(\tau_d + \tau_s)$$

with the results of numerical integrations. It is seen that there is good agreement between the two.

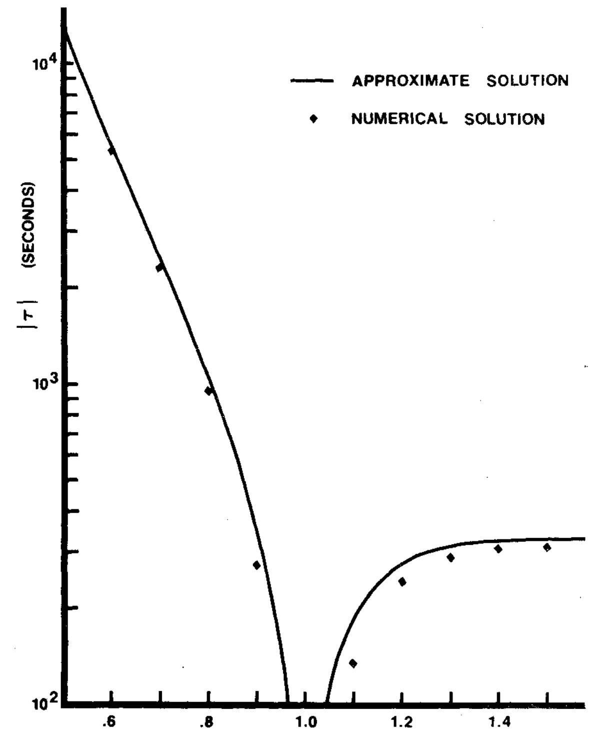

Fig. 7 Approximate vs exact values of τ.

Appendix: Equations of Motion

It is desirable to avoid the complexity inherent in describing the motions of the fluid particle and the damper mass with respect to an inertial frame of reference. This is accomplished by choosing the reference frame to be those axes which would be the principal axes, located at the center of mass, for the system in which $r = z = 0$. For the purposes of this study, it is assumed that the satellite is rotating in free space with zero linear momentum.

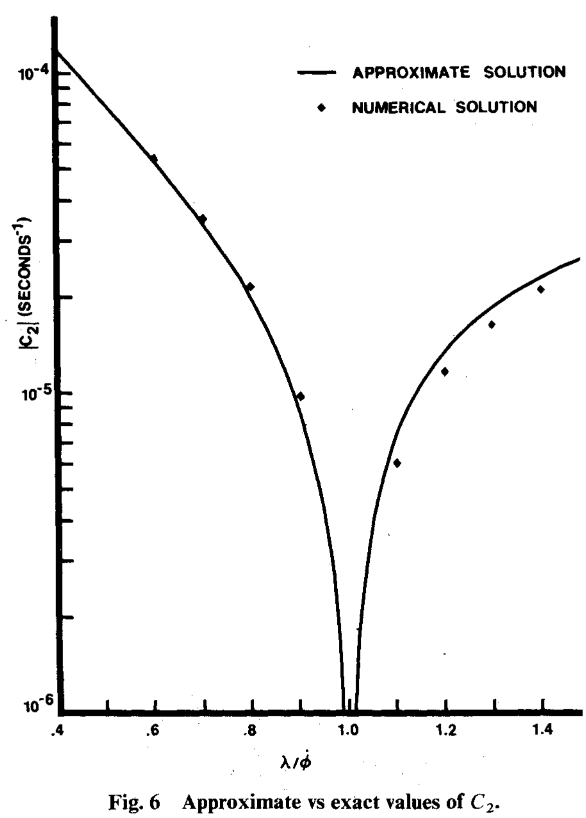

Fig. 6 Approximate vs exact values of C_2.

Since, in general, the origin of the system of axes is in motion with respect to the center of mass of the satellite, the total angular momentum of the system is

$$\mathbf{H} = \mathbf{H}_o - \boldsymbol{\rho}_c \times M\dot{\boldsymbol{\rho}}_c$$

where $\mathbf{H}_o$ is defined as the angular momentum observed by a nonrotating observer translating with the origin of the system, and $\boldsymbol{\rho}_c$ is the vector from the origin of the coordinate system to the center of mass. The momentum $\mathbf{H}_o$ is expressed as

$$\begin{aligned}\mathbf{H}_o = {} & \{I_x\omega_x + M[\mu_d(z^2\omega_x - bz\omega_z) + \mu_s r^2\omega_x \sin^2\psi - \\ & \mu_s r^2\omega_y \sin\psi\cos\psi - \mu_s rl(\omega_z + \dot{\psi})\cos\psi]\}\mathbf{e}_x + \\ & \{I_y\omega_y + M[\mu_d(z^2\omega_y - b\dot{z}) + \mu_s r^2\omega_y\cos^2\psi - \\ & \mu_s r^2\omega_x\sin\psi\cos\psi - \mu_s rl(\omega_z + \dot{\psi})\sin\psi]\}\mathbf{e}_y + \\ & \{I_z\omega_z + I_r\dot{\phi} + M[-\mu_d bz\omega_x + \mu_s r^2(\omega_z + \dot{\psi}) - \\ & \mu_s rl(\omega_x\cos\psi + \omega_y\sin\psi)]\}\mathbf{e}_z\end{aligned}$$

and the position vector $\boldsymbol{\rho}_c$ is

$$\boldsymbol{\rho}_c = \mu_s r\cos\psi\,\mathbf{e}_x + \mu_s r\sin\psi\,\mathbf{e}_y + \mu_d z\,\mathbf{e}_z$$

The terms $\mu_s Ml^2\omega_x\mathbf{e}_x$, $M\omega_y(\mu_s l^2 + \mu_d b^2)\mathbf{e}_y$, and $\mu_d Mb^2\omega_z\mathbf{e}_z$ do not appear in $\mathbf{H}_o$ because, by the definitions of I_x, I_y, and I_z, they are included in $I_x\omega_x\mathbf{e}_x$, $I_y\omega_y\mathbf{e}_y$, and $I_z\omega_z\mathbf{e}_z$, respectively. The total angular momentum is therefore

$$\mathbf{H} = H_x\mathbf{e}_x + H_y\mathbf{e}_y + H_z\mathbf{e}_z$$

where the components are

$$\begin{aligned}H_x = {} & I_x\omega_x + M\{\mu_d[z^2\omega(1-\mu_d) - bz\omega_z] + \\ & \mu_s(1-\mu_s)r^2\omega_x\sin^2\psi - \mu_s(1-\mu_s)r^2\omega_y\sin\psi\cos\psi - \\ & \mu_d\mu_s r\dot{z}\sin\psi + \mu_s r(\mu_d z - l)(\omega_z + \dot{\psi})\cos\psi\}\end{aligned}$$

$$\begin{aligned}H_y = {} & I_y\omega_y + M\{\mu_d[z^2\omega_y(1-\mu_d) - b\dot{z}] + \\ & \mu_s(1-\mu_s)r^2\omega_y\cos^2\psi - \mu_s(1-\mu_s)r^2\omega_x\sin\psi\cos\psi + \\ & \mu_d\mu_s r\dot{z}\cos\psi + \mu_s r(\mu_d z - l)(\omega_z + \dot{\psi})\sin\psi\}\end{aligned}$$

$$\begin{aligned}H_z = {} & I_z\omega_z + I_r\dot{\phi} + M\{-\mu_d bz\omega_x + \mu_s(1-\mu_s)r^2(\omega_z + \dot{\psi}) + \\ & \mu_s r\omega_y(\mu_d z - l)\sin\psi + \mu_s r\omega_x(\mu_d z - l)\cos\psi\}\end{aligned}$$

Conservation of angular momentum yields

$$\dot{\mathbf{H}} = 0$$

which, when broken into its vector components, results in three equations of motion

$$0 = I_x \dot{\omega}_x + (I_z - I_y)\omega_y \omega_z + I_r \dot{\phi}\omega_y + M\{\mu_d(1-\mu_d)[2z\dot{z}\omega_x + z^2(\dot{\omega}_x - \omega_y\omega_z)] - \mu_d bz(\omega_x\omega_y + \dot{\omega}_z) + \mu_s(1-\mu_s)r^2[\omega_y(\omega_z + 2\dot{\psi}) + \omega_x]\sin^2\psi + \mu_s(1-\mu_s)r^2[\omega_x(\omega_z + 2\dot{\psi}) - \dot{\omega}_y]\sin\psi\cos\psi + \mu_s r[(\mu_d z - l)\omega_y^2 - (\mu_d z - l)(\omega_z + \dot{\psi})^2 - \mu_d \ddot{z}]\sin\psi + \mu_s r(\mu_d z - l)(\omega_x\omega_y + \omega_z + \ddot{\psi})\cos\psi\}$$

$$0 = I_y \dot{\omega}_y + (I_x - I_z)\omega_x \omega_z - I_r \dot{\phi}\omega_x + M\{\mu_d(1-\mu_d) \times [2z\dot{z}\omega_y + z^2(\dot{\omega}_y + \omega_x\omega_z)] + \mu_d b[z(\omega_x^2 - \omega_z^2) - \ddot{z}] + \mu_s(1-\mu_s)r^2[\dot{\omega}_y - \omega_x(\omega_z + 2\dot{\psi})]\cos^2\psi - \mu_s(1-\mu_s)r^2[\dot{\omega}_x + \omega_y(\omega_z + 2\dot{\psi})]\sin\psi\cos\psi + \mu_s r(\mu_d z - l)(\dot{\omega}_z - \omega_x\omega_y + \ddot{\psi})\sin\psi + \mu_s r[(\mu_d z - l)(\omega_z + \dot{\psi})^2 - (\mu_d z - l)\omega_x^2 + \mu_d \ddot{z}]\cos\psi\}$$

$$0 = I_z \dot{\omega}_z + (I_y - I_x)\omega_x \omega_y + I_r \ddot{\phi} + M\{\mu_d b[z(\omega_y\omega_z - \dot{\omega}_x) - 2\dot{z}\omega_x] + \mu_s(1-\mu_s)r^2(\dot{\omega}_z + \ddot{\psi}) + \mu_s(1-\mu_s)r^2\omega_x\omega_y[\cos^2\psi - \sin^2\psi] + \mu_s(1-\mu_s)r^2(\omega_y^2 - \omega_x^2)\sin\psi\cos\psi + \mu_s r[2\mu_d\dot{z}\omega_y + (\mu_d z - l)(\omega_x\omega_z + \dot{\omega}_y)]\sin\psi + \mu_s r[2\mu_d\dot{z}\omega_x + (\mu_d z - l)(\dot{\omega}_x - \omega_y\omega_z)]\cos\psi$$

Applying Newton's second law to the motion of the mass-spring damper and to the motion of the fluid particle yields two more equations of motion

$$0 = M(1-\mu_d)\ddot{z} + (c_d/\mu_d)\dot{z} + [(k/\mu_d) - M(1-\mu_d)(\omega_x^2 + \omega_y^2)]z - M\mu_s r[\dot{\omega}_x + \omega_y(\omega_z + 2\dot{\psi})]\sin\psi + M\mu_s r[\dot{\omega}_y - \omega_x(\omega_z + 2\dot{\psi})]\cos\psi + Mb(\omega_x\omega_z - \dot{\omega}_y)$$

$$0 = M(1-\mu_s)r(\dot{\omega}_z + \ddot{\psi}) + (c_s/\mu_s)r(\dot{\psi} - \dot{\phi}) + M(1-\mu_s)r[\omega_y\cos\psi - \omega_x\sin\psi][\omega_y\sin\psi + \omega_x\cos\psi] + M[(\mu_d z - l)(\dot{\omega}_y + \omega_x\omega_z) + 2\mu_d\dot{z}\omega_y]\sin\psi + M[(\mu_d z - l)(\dot{\omega}_x - \omega_y\omega_z) + 2\mu_d\dot{z}\omega_x]\cos\psi$$

Applying $\mathbf{M} = \dot{\mathbf{H}}$ to the rotor yields the sixth and final equation of motion

$$0 = I_r(\dot{\omega}_z + \ddot{\phi}) + c_s r(\dot{\phi} - \dot{\psi}) - T_r$$

where T_r is the torque between the main body of the satellite and the rotor.

References

[1] Carrier, G. F. and Miles, J. W., "On the Annular Damper for a Freely Precessing Gyroscope," *Journal of Applied Mechanics*, June 1960, pp. 237–240.

[2] Cartwright, W. F., Massingill, E. C., and Trueblood, R. D., "Circular Constraint Nutation Damper," *AIAA Journal*, Vol. 1, No. 6, June 1963, pp. 1375–1380.

[3] Cartwright, W. F., Massingill, E. C., and Trueblood, R. D., "Circular Constraint Nutation Damper Analysis," TM 62-205, 1961, General Motors Corp., Detroit, Mich.

[4] Alfriend, K. T., "Analysis of a Partially Filled Viscous Ring Damper," TM X-732-71-456, Oct. 1971, NASA.

[5] Likins, P. W., Tseng, G., and Mingori, D. L., "Stable Limit Cycles due to Nonlinear Damping in Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 8, No. 6, June 1971, pp. 568–574.

[6] Alfriend, K. T., "The Partially Filled Viscous Ring Damper," *Journal of Spacecraft and Rockets*, Vol. 11, No. 7, July 1974, pp. 456–462.